Collider Physics Homework Project 1: Decays

Choose and do ONE of the following.

1. Muon Decay

A muon (μ^-) is a heavier version of the electron (e^-). It decays through the process $\mu^- \to \nu_\mu e^- \nu_e^*$, which is mediated by a W boson, a massive charged (complex) vector particle responsible for mediating the weak nuclear force. We will treat the muon, electron, and the two neutrinos (ν_μ and ν_e) as complex scalar particles (they are fermions), but the W boson correctly. The interaction Lagrangian is

$$
\begin{aligned}
\mathcal{L}_{int} \;=\; & ig\left\{W_\mu^+ \left(\mu^* \partial^\mu \nu_\mu - \nu_\mu \partial^\mu \mu^* + e^* \partial^\mu \nu_e - \nu_e \partial^\mu e^*\right)\right. \\
& \left. + W_\mu^- \left(\mu \partial^\mu \nu_\mu^* - \nu_\mu^* \partial^\mu \mu + e \partial^\mu \nu_e^* - \nu_e^* \partial^\mu e\right)\right\}
\end{aligned}
\tag{1}
$$

where g is a dimensionless coupling constant. We can derive the same form of the vertex we had for scalar QED to describe the W-μ-ν_μ and W-e-ν_e interactions. (You don't need to repeat the derivation, just write it down). You will also need the Massive W propagator carrying momentum p to proceed:

$$
\frac{i(-g_{\mu\nu} + p_\mu p_\nu / M_W^2)}{p^2 - M_W^2 + i\epsilon}
\tag{2}
$$

Write down the Feynman diagram which describes the decay and evaluate it. Simplify it in the limit in which the electron and both neutrino masses are zero, but keep the muon mass. Expand your resulting amplitude in a power series in m_μ / M_W and keep only the leading term. Compute the decay width in that limit.

2. $\pi^0 \to \gamma\gamma$

A neutral pion (π^0) is a neutral (real) scalar particle that arises as a bound state of a quark and an anti-quark. It decays into two photons through the interaction:

$$
\mathcal{L}_{int} \;=\; B \quad \pi^0 F^{\mu\nu} F^{\sigma\lambda} \epsilon_{\mu\nu\sigma\lambda}
\tag{3}
$$

where B is a real coupling constant, $F^{\mu\nu}$ is the electromagnetic field strength tensor, and $\epsilon_{\mu\nu\sigma\lambda}$ is the totally anti-symmetric tensor. What units does B carry? Is this vertex gauge invariant? - Explain your answer. Derive the Feynman rule for this vertex carefully, and use it to compute the decay width of the π^0.

3. Higgs Boson Decays

The Higgs boson (h^0) is a neutral scalar particle that provides mass to all of the other particles in the Standard Model. As a result, its couplings are proportional to the mass of the particle it interacts with:

$$
\mathcal{L}_{int} \;=\; h^0 \left(A_f f^* f + \frac{A_Z}{2} Z_\mu Z^\mu + A_W W_\mu^+ W_-^\mu\right)
\tag{4}
$$

with $A_f = m_f$, $A_Z = M_Z$, and $A_W = M_W$. We will neglect the lightest particles of the Standard Model, because they couple very very weakly and thus the Higgs does not usually decay into them. From Equation (4), write down the Feynman rules for the Higgs coupling to Z bosons, W bosons, and a matter particle f. Compute the partial width for a Higgs decaying into bottom quarks $f = b$, $m_b = 5$ GeV and top quarks $f = t$, $m_t = 175$ GeV. You will have to multiply your final answer for the partial width into quarks by 3, to account for the fact that there are three colors of quarks and the Higgs can decay into any one of them. Also compute the partial width into $W^+ W^-$ and ZZ ($M_Z = 90$ GeV and $M_W = 80$ GeV), summing over the polarizations of the final state vector particles. Don't try to do that polarization by polarization, instead square the amplitude and use the relation (for a massive vector of mass M carrying momentum p):

$$
\sum_\lambda \epsilon_\lambda^\mu \epsilon_\lambda^{*\nu} = -g^{\mu\nu} + p^\mu p^\nu / M^2
\tag{5}
$$

Compute the total width and branching ratios for a Higgs of mass 120 GeV, 200 GeV, and 600 GeV.